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Energy only, capacity market and security of supply.
A stochastic equilibrium analysis

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Energy only, capacity market and security of supply. A stochastic equilibrium analysis

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Abstract

Former generation capacity expansion models were formulated as optimization problems. These included a reliability criterion and hence guaranteed security of supply. The situation is different in restructured markets where investments need to be incentivised by the margin resulting from electricity sales after accounting for fuel costs. The situation is further complicated by the payments and charges on the carbon market. We formulate an equilibrium model of the electricity sector with both investments and operations. Electricity prices are set at the fuel cost of the last operating unit when there is no curtailment, and at some regulated price cap when there is curtailment. There is a CO2 market and different policies for allocating allowances. Todays situation is quite risky for investors. Fuel prices are more volatile than ever; the total amount of CO2 allowances and the allocation method will only be known after investments has been decided. The equilibrium model is thus one under uncertainty. Agents can be risk neutral or risk averse. We model risk aversion through a CVaR of the net margin of the industry. The CVaR induces a risk neutral probability according to which investors value their plants. The model is formulated as a complementarity problem (including the CVaR valuation of investment). An illustration is provided on a small problem that captures the essence of today electricity world: a choice restricted to coal and gas, a peaky load curve because of wind penetration, uncertain fuel prices and an evolving carbon market (EU-ETS). We show that we might have problem of security of supply if we do not implement a capacity market.

Keywords: capacity adequacy, risk functions, stochastic equilibrium models

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1 Introduction

It is now about half a century that optimization models of generation capacity expansion appeared in electrical engineering. The domain took advantage of the development of most mathematical programming techniques and improved both the representation of the power system and the computational performance of the models. This led to better descriptions of machine characteristics, to the inclusion of system criteria such as reliability or the inclusion of global economic characteristics like uncertainties on fuel prices, or demand or appearance of new technologies affecting these major investments. The interest in capacity expansion models came to an almost complete halt with the restructuring of electricity systems. The idea that competition now drives investments led many to draw the conclusion that planning like models are now obsolete and should be replaced by standard investment analysis or by more or less heroic adaptations of financial models. It is fair to say that these developments have so far not provided the comforting solidity of the old capacity expansion models operated in the former regulated environment. But there might be different reasons, other than technicalities, that explain this unsettling feeling.

While the recourse to investment models in the regulated system almost automatically assured that the necessary capacity would develop to keep the lights on, there is today a general concern of resource adequacy. Except for a general faith in the virtues of the market, we are not yet quite certain as to the exact mechanisms that induce investments in a competitive electricity system. The problem is not facilitated by the uncertainties of all sorts that accumulate on the sector. They will not completely hinder new investments, but may reduce their extent. There is certainly considerable investment in new capacities today. But this comes after a long period of sluggish development that led to a reduction, and today to a saturation of the capacities of equipments providers. What determines security of supply is the total capacity, not the rate at which one adds new one after a period of low investment. Studies indeed indicate not only a need to replace ageing fleets but also to adapt them to the challenge of climate change, an area where there is also considerable uncertainty as to future policy. This paper is an attempt to capture, admittedly in a very simplified model, two of current concerns: do we understand the forces driving investments and can we capture the impact of uncertainties?

Resource adequacy was not a problem during the command and control days of regulation. It is today a more delicate issue that we try to tackle through mechanisms such as capacity payments or markets, reliability pricing or other means. We concentrate in this paper on a comparison of capacity and energy only markets. Because we are now in a competitive environment, we move from the former optimization paradigm and formulate this problem as an equilibrium model. This means that we expand the scope of analysis from exogenous costs to endogenous prices. Uncertainty has now reached an unprecedented level in the power industry, and its impact on investment cannot be handled by quietly stating that the market will take care of everything. Uncertainty about the evolution of fuel prices is certainly as large today as it was during the two oil crisis of the seventies. Climate change is a new risk factor, not only due to the very nature of the problem, but also because of the uncertainties about the policies implemented to mitigate its development and effect . Specifically, the European Emission Trading System (EU-ETS) moves a lot of uncertainties about climate change to the power sector, as the latter is conveniently meant to adapt to whatever policy public authorities think relevant to cope with the problem. We accommodate the problem by casting our equilibrium model in a context that explicitly includes a representation of the revelation of information on climate change policy.

The traditional mathematical programming way of dealing with uncertainty is to work with expectations of costs or profits. The crisis of Collateralized Debt Obligation in the subprime market has recently shown the danger of simply assuming away the risk premium or supposing periods of low risk premia in risky markets. Plain expectations are not adequate for investments subject to common cause risks like a tighteninng of emission constraints or an increase of gas prices. The inception of techniques from finance in the energy sector have revealed the importance of risk trading in the short to medium term. But financial markets do not trade long term energy risks such as those affecting new fossil resources or climate change policies. We therefore resort to relatively recent financial and mathematical programming developments and model investor's behavior towards risk by risk functions. In contrast with the existing mathematical programming literature though, we do this in an equilibrium model. This means that we do not assume but derive the risk neutral probability used for assessing investments and the corresponding risk premium from the model. In short we attempt to accommodate to the novel situation by a novel mix of existing techniques.

The paper is structured as follows. Section 2 recalls the standard format of a capacity expansion optimization model that forms the basis of our discussion. That model differs from usual formulations by already encompassing a skeleton representation of the EU-ETS. This immediately forces us to move from optimization to equilibrium. We then introduce the notion of capacity and energy only markets that have been central to the discussion of resource adequacy both in the literature and in practice. Our presentation extensively relies on Joskow (2007), which provides an illuminating discussion of the problem. We develop two versions of the equilibrium model respectively corresponding to the energy only and capacity markets. This is exposed in Section 3. The power sector is today striving in a world plagued by considerable uncertainties. Both stochastic programming and finance theory call upon the notion of stochastic process to represent that uncertainty. An event tree is the simplest possible representation of a stochastic process. We briefly discuss some features of the EU-ETS and cast fuel and ETS uncertainties in a standard two-stage event tree in Section 4. We also indicate how the two deterministic equilibrium models (capacity market and energy only models) are adapted to this stochastic environment. Last we take up the question of behavior towards risk. The finance literature has since a decade developed very powerful representations of attitudes towards risk. These have been adapted to stochastic programming in a rapidly developing literature. We take stock of this literature and endow our equilibrium models with risk behavior represented by a now standard risk function. This is done in Section 5. Section 6 presents some numerical results and discusses the insight that they provide. These are voluntarily limited to express a few messages that we believe may illustrate the relevance of the model. They can be summarized in three sentences. The first message is known and can be traced to the work of several authors: a restructured electricity market that only relies on price signals coming from the sole Power Exchange without any intervention of the Regulator is plagued by an incontrovertible market failure that damages security of supply. The two, possibly new, fundamentally obvious but apparently overlooked messages, are the following. Increasing risk (and the corresponding increase of risk premia) degrades the economics of the system. It also enhances the consequences of the market failure. While our models may look quite distant from standard preoccupations of finance and investment theory, we give them a financial and investment theory (CAPM) interpretation in Section 7.1. A stochastic program may look like a rather rigid economic model; we argue in Section 7.2 that the stochastic equilibrium model constructed with risk functions considerably expands the scope of possible economic descriptions. Specifically, we consider two views of the application of the CAPM models found in practice and show how they can both be cast in our formalism: specifically, we look at whether projects should be valued at their own cost of capital or at the one of the company. We also report that the more natural approach behaves numerically better. Many things have been neglected in this paper: we collect some of them in a brief discussion presented in the conclusion. Specifically, we allude to the new uncertainties brought about by the evolution of the EU-ETS. Our model, very much in the spirit of the first stochastic programming and finance models, is a two-stage problem. We briefly discuss questions related to the extension to multistage problems. For the sake of readability, we conduct all the discussion on an example, but state the relations of the model in general algebraic terms.

2 Reminder: a simple capacity expansion optimization model

We consider the simplest version of a capacity expansion model of an electricity system. The problem is two-periods. One invests in capacities of different technologies in period 0 and operates these capacities in period 1. The example of this paper refers to three technologies, Coal, Combined Cycle Gas Turbine (CCGT) and Open Cycle Gas Turbine (OCGT). The model is general though and not limited to these three technologies as we show when stating it in algebraic form. Each equipment has both an investment and operating cost. The investment costs are in thousand euro per MW; they are computed from overnight construction and fixed operating cost using a standard annualisation procedure (see Table 1).

	Coal	CCGT	OGGT
I: annual capacity and fixed operating cost (k €/Mw)	160	80	60
e: emission t/Mwh	1	.35	.6

Table 1: Fixed annual cost and emission in a three technology world

We come back to the question of the discount rate in Section 7.1 but note here that annuities should be understood as computed with a risk free rate. Because we want to include a representation of a CO_2 market, we also introduce emission factors for each plant. These are in tons of CO_2 emitted per Mwh produced. The operating costs are derived from fuel prices using plant efficiencies. They will be given in Section 4.

The tradition of capacity expansion model is to suppose a price insensitive demand represented by a load duration curve. This is also the approach adopted in Joskow 2007's discussion of resource adequacy. We follow suit in this paper, even though it is no way required by our analysis. The difficulty of going to a price sensitive demand lies much more in the availability of a reasonable demand function of electricity than in the inclusion of that demand function in a capacity expansion model or in any of the extensions considered later in the paper. In the absence of such reliable demand function, we simply assume a price insensitive load duration curve. We segment it in as few demand blocks as possible in order to keep the model simple, while still guaranteeing sufficient detail for arriving at meaningful results. Table 2 gives the relevant figures.

power level and utilisation						
d: MW	86000	83000	80000	60000	40000	20000
τ: duration (1000 hours)	.01	.04	.31	4.4	3	1

Table 2: load duration curve and its decomposition in time segments

The load duration curve is decomposed in 5 time segments, with their length measured in 1000 hours. The three highest demand segments and the lowest segment reveal a rather peaky pattern that is intended to represent the impact of the penetration of wind energy as required by EU objective of 20 % of renewable energy by 2020 (check “Renewable energy in the European Union” in Wikipedia). Wind power indeed reduces the utilization of the peak (ratio of peak demand and energy) but not its level. This justifies working with both a comparatively high load demand in the left of the load duration diagram and the low level in the right. We now formally generalize this simple example to accommodate an arbitrary number of plants and time segments. Notations are given in Tables 3, 4 and 5, for the general model and adapted in this section to a deterministic set up.

The deterministic model of this section should be understood as comprising one fuel (price) scenario, one NAP scenario (total amount of allowances available to the sector or total constraint on CO_2 emission by the sector) and one allowance allocation mode scenario (here taken as full auctioning: plants have to buy as many allowances as the number of tons that they emit). The sets F, N and B of Table 3 therefore each contain only a single element in the deterministic model. Table 4 specializes as follows to the deterministic model. I, d, τ and e already appeared in Tables 1 and 2. Their interpretation is recalled. $a(K, B)$ designates an amount of free allowances received by an investor building a plant k in allocation scenario b. This is expressed in tons of CO_2/year per installed MW. As explained in the beginning of this paragraph, this allocation $a(K, B)$ represents the amount of free allowances, is first set at zero in the deterministic model when we assume full auctioning. The probabilities $pf(F), pn(N)$ and $pb(B)$ are all one in the deterministic model. There is no coefficient of risk aversion α associated to the CVaR (see Section 5 for a discussion of that notion) in the deterministic model. The price cap PC will be explained after stating the model. The list of variables given in Table 5 also applies to all models but is here particularized for the deterministic case. We shall see in Section 5 that the variables $\phi(K; F, N, B)$ have an interpretation of probabilities. There is thus a single ϕ in the deterministic model and its value is 1. There is no Value at Risk (VaR) in the deterministic model. The interpretation of the other variables will become clear as we proceed through the model. Given these particularizations we now set the capacity expansion model as follows.

$\ell \in L$	demand segments
$f \in F$	fuel scenarios
$n \in N$	Nap scenarios
$b \in B$	allocation scenarios
$k \in K$	plant type

Table 3: Sets

$I(K)$	investment costs + maint annualy
$d(L)$	demand
$\tau(L)$	duration
$e(K)$	emission level
$a(K, B)$	allocation by technology and scenario
$pf(F)$	probabilities
$pn(N)$	probabilities
$pb(B)$	probabilities
$NAP(n)$	CO_2 system cap
$c(K, F)$	fuel costs
PC	price cap
α	risk aversion

Table 4: Parameters

$x(K)$	capacity
$y(K, L, F, N, B)$	production
$z(L, F, N, B)$	shortage
$\mu(K, L, F, N, B)$	marginal value of capacity
$\pi(L, F, N)$	price
$\lambda(F, N, B)$	allowance price
ν	marginal value of capacity in the forward mark
$u(K; F, N, B)$	value of the loss with respect to VAR
$\phi(K; F, N, B)$	risk neutral probability
$VaR(K)$	VaR

Table 5: Full set of variables (that is also including equilibrium variables)

We comply with standard practice of stochastic programming and present the second and first stage problems of our two stage model separately. The second stage represents the operation of the power system for given capacities $x(k)$. The problem is written in optimization form and is analoguous to the standard optimal dispatch model. The inclusion of the CO_2 emission constraint might be the only difference between this model and the standard economic dispatch. The dual variables appear at the right of the constraint. Recall that indices f, n, b are dropped in this deterministic (single scenario) model.

$$Q(x) \equiv \min_{y,z} \sum_{\ell \in L} \tau(\ell) \left[\sum_{k \in K} c(k, f)\, y(k, \ell) + PC\ z(\ell) \right] \tag{1}$$

s.t.

$$0 \leq x(k) - y(k, \ell) \qquad \mu(k, \ell) \tag{2}$$

$$0 \leq \sum_{k \in K} y(k, \ell) + z(\ell) - d(\ell) \qquad \pi(\ell) \tag{3}$$

$$0 \leq NAP - \sum_{\ell \in L} \tau(\ell) \sum_{k \in K} e(k) y(k, \ell) \qquad \lambda \tag{4}$$

$$0 \leq y(k, \ell). \tag{5}$$

The interpretation of the short run model is straightforward except possibly for relation (4) that we discuss in more detail. The objective function (1) is the sum of the operating costs over the different plants and time segments. This latter comprises the shortage cost (when demand is not met) here taken as PC which is regulated. It is indeed always possible, in the model, to curtail demand in any time segment ℓ and scenario by reverting to $z(\ell)$. The constraint (2) states that the operation of plant k can never exceed the existing capacity $x(k)$. Constraint (3) expresses that generation plus shortage must be at least equal to demand in each time segment ℓ. Last constraint (4) states that the total emission over the year does not exceed the total amount of allowances NAP. This total is determined by the National Allocation Plans introduced in Directive 2003/87/EC (check "European Union Emission Trading Scheme" in Wikipedia). It suffices for our purpose to note that this amount is fixed exogenously. The total emission is then computed by summing the production of CO_2 over all time segments. The dual variable λ of constraint (4) has the crucial interpretation of allowance price.

Applying standard duality theory, we convert the optimization problem into complementarity form. It is this form, generalized to the case of several scenarios, that we shall refer to in the following when discussing the equilibrium model. Relations (6), (7) and (8) are the complementary slackness conditions of the primal problem (1) to (5). Relations (9) and (10) are the complementary slackness conditions of its dual.

$$0 \leq x(k) - y(k,\ell) \perp \mu(k,\ell) \geq 0 \tag{6}$$

$$0 \leq \sum_{k \in K} y(k,\ell) + z(\ell) - d(\ell) \perp \pi(\ell) \geq 0 \tag{7}$$

$$0 \leq NAP - \sum_{\ell \in L} \tau(\ell) \sum_{k \in K} e(k)\, y(k,\ell) \perp \lambda \geq 0 \tag{8}$$

$$0 \leq c(k) + \mu(k,\ell) + e(k)\lambda - \pi(\ell) \perp y(k,\ell) \geq 0 \tag{9}$$

$$0 \leq PC - \pi(\ell) \perp z(\ell) \geq 0. \tag{10}$$

The complementarity condition (10) clearly shows that the electricity price is capped by PC. This relation will play a crucial role as we discuss the capacity and energy only markets in Section 3.

The capacity expansion problem is written in optimization form as

$$\min_{x \geq 0} \sum_{k \in K} I(k)\, x(k) + Q(x). \tag{11}$$

Noting that $\sum_{\ell \in L} \tau(\ell)\mu(k,\ell)$ is a subgradient of $Q(x)$ with respect to x, one can easily check that (11) can be restated in complementarity form as

$$0 \leq I(k) - \sum_{\ell \in L} \tau(\ell)\, \mu(k,\ell) \perp x(k) \geq 0. \tag{12}$$

The economic interpretation of (12) is straightforward: one invests when the investment cost of the plant is equal to the marginal value of the capacity. One does not invest when it is larger.

We now move from optimization to equilibrium in order to account for some features of the carbon market (EU-ETS). Consider the following slight modification of (12) stated as follows:

$$0 \leq I(k) - a(k)\,\lambda - \sum_{\ell \in L} \tau(\ell)\, \mu(k,\ell) \perp x(k) \geq 0 \tag{13}$$

where $a(k)\,\lambda$ is the value of free allowances received by plant k. Note that (12) and (13) are equivalent when $a(k)$ are zero. But they differ otherwise. Relation (13) can be interpreted as follows: one invests when the investment cost of the plant is equal to the sum of the value of allowances received free and the marginal value of the capacity. This modification is mathematically important: the combination of (6) to (10) and (13) is a complementarity problem (or equilibrium model) which is no longer equivalent to an optimization model.

3 Resource adequacy: capacity markets and energy only markets

The restructuring of the electricity sector began in a context of overcapacity. Reserves were generally high with this high level sometimes considered as the signal of an inadequate regulation that induced excessive investments. The progressive decline of these reserves was initially considered as the beneficial result of restructuring until one began wondering whether it did not instead reflect a market failure. Maybe the restructured market was not sending the right investment signal! Stoft, in his book (Stoft, 2002), is probably the first one to have provided an extensive discussion of the problem of generation investments in restructured systems. The subject has since seen an explosion of interest in both the literature and in practice (see in particular Hogan (2005), Oren (2005) and Cramton and Stoft (2006)). We here rely on the discussion of Joskow (2007) that offers a particularly insightful discussion of the subject.

Joskow refers to the operation of a competitive Power Exchange, which under the assumption that there is no market power remunerates each operating plant at the short run marginal cost, here taken as the fuel cost, of the most expensive operating unit. More specifically, plants are operated in merit order in a competitive system, and the plant of highest fuel cost set the price of electricity, except if the whole capacity of the generation system is utilized, in which case there is a scarcity rent. The phenomenon needs to be slightly adapted to reflect the emissioning constraint and the trade of allowances: plants are still operated in merit order but the short run marginal cost now reflects both the fuel cost and the value of the additional allowances $e(k)\lambda$ that have to be surrendered because of a marginal increase of the functioning of the plant. Again there is a scarcity rent when all the capacity of the generation system is utilized. This phenomenon is readily observed in complementarity conditions (6) to (9). Relation (6) defines the capacity rent $\mu(k, \ell)$ of plant k in time segment ℓ. Relation (9) states that the electricity price $\pi(\ell)$ is smaller or equal to the sum of the fuel cost $c(k, \ell)$, the emission value $e(k)\lambda$ and the scarcity rent $\mu(k, \ell)$ when the plant k does not operate in time segment ℓ. It is equal when plant k operates in time segment ℓ.

The missing element of the pricing scheme expressed in relations (6) to (9) is how the price of electricity $\pi(\ell)$ and hence the value of the scarcity rent is determined when all capacity is used up: there is no further plant to set the price. The standard reasoning in a perfect competition market is to assume a downward sloping curve that intersects the capacity limit and thus sets the price at the intersection with the vertical of full capacity: the scarcity rent rations the demand so as to make it compatible with the existing capacity. This is depicted on Figure 1a. The reality is that the short term demand function of electricity is generally considered to be inelastic implying that both demand and the vertical line may not intersect (Figure 1b). The result is that one can determine neither the price of electricity nor the scarcity rent. This is the origin of the market failure initially described in Stoft (2002).

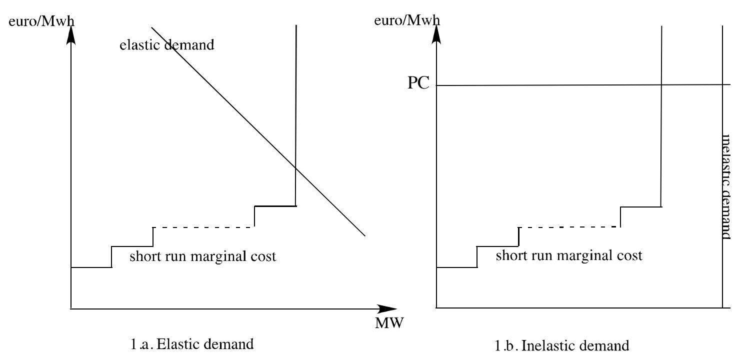

Figure 1: Price setting when capacity is used up

The solution to this problem in energy only system is to price electricity at a high value that is supposed to reflect the value of lost load (VOLL) when demand is curtailed. This is also represented on Figure 1b where PC has been set at VOLL. A more sophisticated method is to apply some form of reliability pricing whereby electricty price increases when reserve margin decreases. An alternative solution to avoid market failure is to implement a capacity market whereby the TSO or the regulator imposes some capacity target in line with expected demand and conducts an auction to procure the needed capacity. This type of system is implemented in New England. None of this has been effectively discussed by European entities in charge of restructuring. One finds trace of these, neither in the work of the European Transmission System Operators (see ETSO website) nor of the European Regulators (see ERGEG website). The Directive 2005/89/E on Security of Electricity Supply mentions the list of these measures but the recital of the Directive is silent on their justification and relative merits. In contrast with the absence of any real treatment of investment incentive mechanism, a considerable attention has been devoted to the possible exercise of market power by companies when capacities are tight: any departure of the price with respect to fuel cost is more readily interpreted in those terms than in scarcity rent and it is the reasoning that is currently prevalent in Europe (see London Economic 2007 conducted for Competition Authorities). The result is that it is very unlikely that Regulator or Competition Authorities would allow companies to charge a full VOLL in case of Lost Load. They would interpret such a high price (VOLL is commonly assumed to be of the order of 10 000 €/Mwh) as excessive. In the absence of any explicit reliability pricing mechanism or capacity market, the only solution is to assume that companies will test the extent to which they can raise prices during tight capacity conditions. Prices will thus increase in order to reflect the scarcity of capacity, but not too much in order for this increase not to be interpreted as an exercise of market power. We thus suppose an implicit regulation of prices at some maximum level PC (Price Cap) that applies in case capacity is insufficient to meet demand. In order to fix ideas we consider two cases of 1000 and 250 euros/Mwh and consider that investments in Europe is implicitly driven by an energy only system where prices during curtailments are capped at PC. It is this system that we want to compare with one that also embeds a capacity market.

To sum up we consider both the cases of a capacity market and of an energy only market where prices are capped at some value PC respectively taken as 1000 and 250 euros/Mwh. The deterministic energy only market is thus described by complementarity relations (6) to (10) and (12). The incentive to invest is driven by the price PC charged when capacity is tight (or insufficient in this simple model). The introduction of a capacity market requires a modification of the long run condition (12), which is now replaced by two conditions. The relation

$$0 \leq \sum_{k \in K} x(k) - \max_{\ell \in L} d(\ell) \perp \nu \geq 0 \tag{14}$$

expresses the capacity target to which one associates a capacity value ν.

$$0 \leq I(k) - a(k) - \nu - \sum_{\ell \in L} \tau(\ell)\, \mu(k) \perp x(k) \geq 0 \tag{15}$$

gives the new investment condition. One easily sees that relation (14) is identical to relation (12) except for the addition of a capacity remuneration ν where this remuneration is the variable associated to the capacity target (13).

4 Risk

The power sector evolved from an almost risk free environment in the days of regulation to one of extreme uncertainty today. We here consider two types of risk that we respectively categorize as external and EU-induced. Fuel price risk is the paradigm of external risk. It comes from world wide movements. EU-induced risks are those arising from EU policies like internal market or environmental policies. We take the EU-ETS as a representative source of EU-induced risk: the EU is leading the fight against climate change by implementing an uncharted version of a “cap and trade”. This system has been successively implemented in the US acid rain program. The EU-ETS transposes it to a much broader context affected by considerably more uncertainties. This creates risk. Both risks combine and are likely to have an inpact on the incentive to invest. This impact is today unknown.

4.1 External risk

Fuel prices and demand are two major risk factors that have accompanied the electricity sector since the first oil crisis in the early seventies. Demand was growing at a rate close to a strong 7 % after the end of the second world war until it began a steady decrease after oil prices, were multiplied by 4 between 1973 and 1974. The market has remained someone in turmoil since then, with a new upsurge of oil prices occurring since 2005. Gas prices, which are indexed on oil and oil products followed suit. The astounding economic growth rate of countries like India and China contributed to the tension in the oil market but also induced a significant increase of the demand for coal. We here adopt a very simplified view of that uncertainty and only consider two scenarios of oil and gas prices, that we assume of equal probability. There are depicted in Table 6.

	scenario f1	scenario f2
coal	30	30
CCGT	45	68
OCGT	80	120
Prob.	.5	.5

Table 6: plant operating cost scenarios (in k€/k hours)

In order to simplify our discussion, we do not include any demand uncertainty in the example treated in this paper.

4.2 EU-induced risk

The growing concern about climate change induced various actions by European authorities to curb GHG emissions in general and CO_2 emissions in particular. Other measures aimed at developing renewable energy indirect complement and support those CO_2 reduction policies. The decision process in a weak federal system such as the EU is very delicate and lengthy. Policies therefore take a considerable time to be devised and implemented. The result is that fundamentally sound objectives dissipate in murky details of implementation. Taking the CO_2 reduction policy, even though the EU-ETS has been introduced in a first Directive in 2003 (Directive 2003/87/EC), the detail of its implementation in the "post Kyoto" period, that is after generation units decided in 2007 come on line, remained largely unknown at the time of the decision. This type of situation creates considerable uncertainties that have an impact on investment decisions. We only consider this CO_2 policy in our example. We first describe a very simple representation of the EU Emission Trading Scheme (EU-ETS) introduced in our test example. The interested reader can consult the website of DG

Environment of the European Commission for more details on the subject. We concentrate on two EU-ETS risk factors, that prevail at the time of this writing.

The EU-ETS is a cap and trade system that imposes firms from different economic sectors to surrender emission allowances for every emitted ton of CO_2. The covered combustion installations are defined in Directive 2003/87/EC. Installations either receive some allowances free and can procure the rest on a market or through an auction. The Directive created a market where allowances can be traded.

Combustion installations can buy allowances to compensate a shortage, or dispose of excess allowances on that market. The Directive has so far provided for a significant fraction of allowances allocated free. Its revision will most likely require auctionning a significant fraction or all of them. Extensive discussions have also taken place as to the best allocation mechanism for the remaining free allowances. The amount of allowances received free is crucial information when it comes to investment. Free allowances are in fact a subsidy. Its value depends on the value of the allowances on the market. But more free allowances will always mean more subsidy. We retain the possibility of both auctioned and free allowances. In the latter case we suppose a distribution either per installed capacity (irrespective of the technology) or benchmarked with respect to Best Available Technology for each type of plant (in this case computed on the basis of a certain number of hours of operations). In all cases, the result is expressed by a certain granting of free allowances a (see Table 4). Full auctionning simply set a to zero. The scenarios corresponding to these policies are represented in Table 7, together with their assumed probabilities.

	scenario b1	scenario b2	scenario b3
coal	6	2.1	0
CCGT	2.1	2.1	0
OCGT	1.2	2.1	0
Prob.	0.1	0.3	0.6

Table 7: scenarios of free allocation by unit of capacity

The higher probability given to the full auctioning scenario reflects the current proposal of the European Commission: it is not certain that it will be accepted as such, but it is very likely that there will be an important shift from free allowances required in the initial 2003 Directive towards full auctioning in the revised one. It should be recalled that the information about the amount of free allowances, very much like the fuel price, is not known at the time of investment in 2007. Plants thus need to be decided not knowing whether they will receive free allowances or not; if they do, they do not know the allocation mode and the amount of allowances.

The other major unknown of the EU-ETS is the total reduction of CO_2 emission required from the covered installations. Leaving aside all questions of "flexibility mechanisms" related to the Kyoto protocol we simplify the discussion and assume that the required reduction is reflected in the total amount of allowances proposed by Member States (the National Allocation Plans or Nap) and eventually accepted, often after reduction, by the Commission. Again the total amount of allowances available, whether free or through an auction, to the covered sectors for after the year 2012 is unknown at the time of investment. We stylize this uncertainty in the scenarios depicted in Table 8.

	scenario n1	scenario n2
in million ton	200 000	240 000
Prob.	0.5	0.5

Table 8: NAP scenarios

While the *NAP*s refer to all combustion plants covered by the EU-ETS, the figures given here should be understood as covering the sole power sector. Because of lack of information, we are indeed unable today to model the global market of allowances. The situation is similar to the one prevailing for the demand for electricity: very much like one lacks robust information on price elastic demand of electricity, one cannot rely today on any reasonable representation of the response of the industrial sectors to allowances prices. The figures of Table 8 have been computed as follows. Starting from emission of the sectors simulated without ETS, emission targets of 20 and 30% have been applied.

4.3 Summing up and modeling implication

The above discussion leads to two price scenarios (set F), 3 allowance allocation modes (set B) and 2 NAP scenarios (set N). This means a total of 12 scenarios that we note by their defining indices (f, n, b). The probabilities of these scenarios are obtained by straight multiplications of $pf(f), pn(n)$ and $pb(b)$, as we assume that events on fuel costs, NAPs and allocations modes are independent. The following complementarity conditions extend relations (6) to (14) that represent a deterministic world to this more general case involving uncertainty. They do so by introducing additional indexing of the variables to reflect the scenarios.

The equilibrium of the short run market is represented as follows:

for all (f, n, b)

$$0 \leq x(k) - y(k, \ell, f, n, b) \perp \mu(k, \ell, f, n, b) \geq 0 \tag{16}$$

for all (f, n, b)

$$0 \leq \sum_{k \in K} y(k, l, f, n, b) + z(\ell, f, n, b) - d(\ell) \perp \pi(\ell, f, n, b) \geq 0 \tag{17}$$

for all n

$$0 \leq NAP(n) - \sum_{\ell \in L} \tau(\ell) \sum_{k \in K} e(k) y(k, \ell, f, n, b) \perp \lambda(\ell, f, n, b) \geq 0 \tag{18}$$

for all (f, n, b)

$$0 \leq c(k, f) + \mu(k, \ell, f, n, b) + e(k)\lambda(f, n, b) - \pi(\ell, f, n, b) \tag{19}$$

$$\perp y(k, \ell, f, n, b) \geq 0$$

for all (f, n, b)

$$0 \leq PC - \pi(\ell, f, n, b) \perp z(\ell, f, n, b) \geq 0. \tag{20}$$

The equilibrium of investment in the energy only market is given as follows:

for all k

$$0 \leq I(k) - \sum_{f \in F, n \in N, b \in B} pb(b) a(k, b) pf(f) pn(n) \lambda(f, n, b)) \tag{21}$$

$$- \sum_{\ell \in L, f \in F, n \in N} \tau(\ell) pb(b) pf(f) pn(n) \mu(k, \ell, f, n, b) \perp x(k) \geq 0.$$

The equilibrium of investment in a capacity market obtains as follows:

$$0 \leq \sum_{k \in K} x(k) - \max_{\ell \in L} d(\ell) \perp \nu \geq 0 \tag{22}$$

for all k

$$\begin{aligned} 0 \leq I(k) - & \sum_{f \in F, n \in N, b \in B} pf(f)pn(n)pb(b)a(k,b)\lambda(f,n,b)) - \nu \\ & - \sum_{\ell \in L, f \in F, n \in N, b \in B} \tau(\ell)pf(f)pn(n)pb(b)\mu(k,\ell,f,n,b) \perp x(k) \geq 0. \end{aligned} \tag{23}$$

5 Attitudes towards risk

5.1 P and Q versions of the investment model

The above discussion only involves risk neutral investors; it describes a market where the risk premium is zero. Relations (16) to (21) refer to the energy only market. The combination of relations (16) to (20) with (22) and (23) gives the capacity market model. In both organizations, investors base their decisions on the sole expected profit that these decisions imply; they do not respond to the profile of these profit scenarios. For instance, investors disregard whether some of these scenarios could lead to bankruptcy or not, provided the total expected reward is the same. We now expand the economic interpretation of the complementarity conditions to represent alternative behaviors towards risk. We first explain relation (21) (energy only model) and relations (22) and (23) (capacity market) in basic economic terms as it is this economic interpretion that we shall modify to introduce risk averse investors. Consider relation (21). It states that one invests in a particular equipment k if the expected gross margin accruing to that equipment is equal to its investment cost. Relation (23) has a similar interpretation after adapting the gross margin to account for the capacity price ν. Gross margins are computed for each scenario (f, n, b) and are equal to the revenue accruing to the equipment in each scenario minus its operating cost. The revenue results from the sale of electricity, of allowances obtained free and from the sale of capacities when there is a capacity market. The costs are those of the consumed fuels and of the allowances surrendered in compensation for emissions. In this model, and following Joskow 2007's description of a competitive Power Exchange, electricity is sold at a price equal to the sum of fuel cost and allowance opportunity cost of the most expensive running equipment, when there remains some spare capacity. Electricity is sold at the price cap when all capacity is used up.

Abusing the language of mathematical finance, we refer to the probability $pb(b)\, pf(f)\, pn(n)$ of scenario (b, f, n) introduced in Section 4 as the "statistical probability" and note it P. Risk neutral investors decide by taking expectations on the basis of this statistical probability. In contrast risk averse investors decide on the basis of a modified probablity that we refer to as the risk neutral probability and note it Q. We observe at this stage that we can formally replace the statistical probability P by any "risk neutral" probability Q in relations (21) or (23) without changing the mathematical nature of the overall model. We thus refer to the P and Q versions of the equilibrium model as the one describing risk neutral and risk averse investors.

The Q and P versions of the equilibrium model have the same mathematical form. The P model represents risk neutral investors. In order to represent risk averse investors, we draw upon the theory of risk functions and suppose that investors behave according to a certain risk function from which we derive a certain risk neutral probability Q. We describe these two steps.

5.2 Risk functions and risk averse investors

Risk functions were introduced in Artzner et al. (1989) pathbreaking paper. The authors later developed this notion in several publications (e.g. Artzner et al. 2002 and 2004). Rockafellar and Uryasev (2002) showed how one of these risk functions namely the CVaR could be cast in an optimization form. Different authors extended these ideas in an optimization context. In so doing, they expanded on a duality theory of risk measures, already present, but not extensively elaborated in Artzner et al. fundamental paper. We construct our measure Q on the basis of that duality theory.

Specifically, suppose that investors no longer reason in expected profits but use the CVaR of the distribution of marginal profits. It may be surprising to rely the CVaR in a profit context as this function is more commonly used for assessing losses, but we shall see that this does not cause any particular difficulty. First note that investors in a perfectly competitive market can only reason in terms of marginal costs, that is in terms of the π, μ and λ. By definition, they are price takers and cannot modify any of these values. They thus value the gain from a marginal investment through marginal margins computed on the basis of these marginal π, μ and λ. Assume now that risk averse investors value new capacities on the basis of the CVaR of the marginal margin or profit accruing from them. This means that investors base their valuation of an investment on the conditional expectation of the profit lower or equal to a threshold profit that is only exceeded α % of the time. The notion is illustrated on Figure 2 for a continuous distribution of profits.

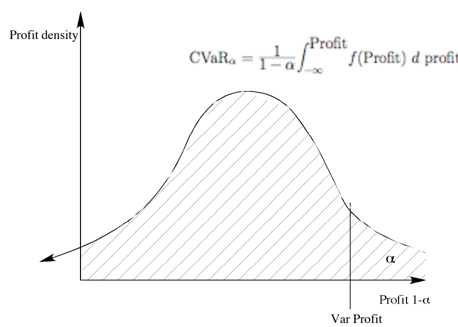

Figure 2: Illustration of the CVaR_α

Starting from a VaR_α level of profits that is only exceeded with probabiliy $\alpha\%$, one decides on the basis of the conditional expected profits lower or equal to this VaR_α level. The problem simplifies in the simple example of a few discrete scenarios presented in this paper. Using 12 scenarios as in the example and taking $\alpha = 10\%$ amounts to discarding the most profitable scenario and computing the conditional expectation on the remaining ones (that is where the probability space is restricted to 11 scenarios). Discarding the most profitable scenarios expresses the risk aversion or the prudence of the investor.

The CVaR is only one of the several risk measures that can be used in this fashion. We retain it in this paper and described its implications in terms of risk neutral probabilities. Any other risk measure in the sense of Artzner et al. (1989) (that is satisfying their four axioms) would lead to similar developments.

5.3 Duality and complementarity representation of risk measures

Artzner et al. (1989) showed that risk measures can be represented in two different ways that are dual to each other. Rockafellar and Uryasev (2002) cast the idea of the CVaR in an optimization context. Eichhorn and Römish (2005) generalize the approach to polyhedral risk functions. Ruszczynski and Shapiro (2006) present a thorough treatment of the use of different risk functions in optimization problems. Going from the CVaR optimization representation to its dual allows one to recover Artzner et al. notion of pseudo scenario and hence the corresponding risk neutral measure. We recall the particularly simple derivation of the CVaR case and embed it in the capacity market model. The treatment of the energy only model is analogous.

The net margin (after substracting investment costs) from an investment in capacity k accruing to investors in scenario (f, n, b) is given by

$$\text{margin}(k; f, n, b) \equiv \sum_{\ell \in L} \tau(\ell)\mu(k; \ell, f, n, b) + a(k, b)\lambda(f, n, b) + \nu - I(k).$$

Rockafellar and Uryasev showed that the CVaR(k) of this profile of net margins generated by plant k is given as the solution of the following maximization with respect to VaR(k).

$$\begin{aligned} \min_{\text{VaR}(k)} \; - \text{VaR}(k) + \tfrac{1}{\alpha} \textstyle\sum_{f \in F, b \in B, n \in N} pf(f)pb(b)pn(n) \\ \max(\text{VaR}(k) - \text{margin}(k; f, n, b), 0). \end{aligned} \tag{24}$$

Rockafellar and Uryasev also showed that the optimal solution of this problem is indeed the Value at Risk (VaR) of the distribution of the net margins. Dropping the index k for the sake of simplicity in the rest of this section, problem (24) can be rewritten in LP form

$$\min_{\text{VaR}} - \text{VaR} + \frac{1}{\alpha} \sum_{f \in F, b \in B, n \in N} pf(f)pb(b)pn(n)u(f, n, b) \tag{25}$$

$$\text{s.t.} \quad u(f, n, b) \geq \text{VaR} - \text{margin}(f, n, b) \quad \phi(f, n, b) \tag{26}$$

$$u(f, n, b) \geq 0 \tag{27}$$

The duality conditions at optimum can be written in complementarity form as

$$
\begin{aligned}
0 &\leq u(f,n,b) - \mathrm{VaR} + \sum_{\ell \in L} \tau(\ell)\mu(\ell,f,n,b) + \nu \\
&+ a(b)\lambda(f,n,b) - I(k) \perp \phi(f,n,b) \geq 0 && (28) \\
0 &= - \sum_{f \in F, b \in B, n \in N} \phi(f,n,b) + 1 && (29) \\
0 &\leq -\phi(f,n,b) + \frac{1}{\alpha} pf(f)pn(n)pb(b) \perp u(f,n,b) \geq 0. && (30)
\end{aligned}
$$

The variables ϕ appearing in these conditions are probabilities as shown by their non negativity and relation (29). They are also the desired risk neutral probabilities. We indeed see from (26) that $\phi(f,n,b) = 0$ and $u(f,n,b) > 0$ when $\mathrm{VaR} > \mathrm{margin}(f,n,b)$. Relation (30) therefore implies

$$
\begin{aligned}
& \sum_{f \in F, b \in B, n \in N} \phi(f,n,b) \mathrm{margin}(f,n,b) \\
= & \frac{1}{\alpha} \sum_{f \in F, b \in B, n \in N} pf(f)pn(n)pb(b)\mathrm{margin}(f,n,b) && (31) \\
\equiv & \mathrm{CVaR}\, R_\alpha(\mathrm{margin}(f,n,b)).
\end{aligned}
$$

In other words, the expectation of the margins is also their CVaR.

5.4 Reformulation of the equilibrium problem for risk averse investors

We can now readility convert the P version of the energy only and capacity market models to a corresponding Q version that represents risk averse, CVaR behaving agents. Expressing that these agents maximize their profit by investing in plant k according to the risk neutral probability $\phi(k; f, n, b))$ derived from the duality contitions of the CVaR(k), we introduce in the model the following conditions that define the risk neutral probabilities $\phi(k; f, n, b)$:

for all $k \in K$

$$0 \leq u(k;f,n,b) - \mathrm{VaR}(k) + \mathrm{margin}(k;f,n,b) \perp \phi(k;f,n,b) \geq 0 \quad (32)$$

$$0 = - \sum_{f \in F, b \in B, n \in N} \phi(k;f,n,b) + 1 \quad (33)$$

$$0 \leq -\phi(k;f,n,b) + \frac{1}{\alpha} pf(f)pn(n)pb(b) \perp u(k;f,n,b) \geq 0 \quad (34)$$

We note that we have as many risk neutral probabilities as the number of plant types. We replace the investment criterion (21) (energy only market) and (23) (capacity market) by the single investment criterion expressed as

$$0 \leq - \sum_{f \in F, b \in B, n \in N} \phi(k; f, n, b) \text{margin}(k; f, n, b) \perp x(k) \geq 0 \tag{35}$$

where margin$(k; f, n, b)$ is equal to

$$\begin{aligned} \text{margin}(k; f, n, b) \quad & \equiv \sum_{\ell \in L} \tau(\ell) \mu(k, \ell, f, n, b) + \nu \\ & + a(k, b) \lambda(f, n, b) - I(k) \end{aligned} \tag{36}$$

for the capacity market and

$$\begin{aligned} \text{margin}(k; f, n, b) \quad & \equiv \sum_{\ell \in L} \tau(\ell) \mu(k, \ell, f, n, b) \\ & + a(k, b) \lambda(f, n, b) x(k) - I(k) \end{aligned} \tag{37}$$

for the energy only market.

The rest of the model (that is relations (16) to (20)) remains unchanged.

6 Numerical results and policy implications

The discussion of the respective merits of energy only and capacity market organisations has been going on since many years in the literature. The two methods have also been implemented in different restructured electricity systems. Needless to say measuring the full impact of these investment incentives is a long term process and one is still waiting for clear cut evidence from the real world. None of this discussion has pervaded European Transmission System Operators (see ETSO website) or European Regulators (see ERGEG website) debates in the EU so far. The result is that European markets are largely energy only, without this resulting from any deliberate choice. The principle of energy only markets is to charge a high electricity price when capacity is tight. Our simple example model inherited from Joskow 2007 does not recognize the notion of tightness: capacity is sufficient or not. This corresponds to the situation discussed in Stoft (2002) where the price of electricity is set to VOLL in case of curtailment. The principle of energy only markets is then to apply a regulated cap when capacity is insufficient. Both Joskow and Stoft argue that the choice of this cap is far from inconsequential. The cap has an impact on the invested capacity in the energy only market; in practice, it changes the reliability of the system. The problem of which cap to choose can be posed in technical terms by calling upon standard notions of reliability such as Loss of Load Probability. It can also be looked at in less technical, possibly more economical and in some cases definitely more political terms of market power.

Examples abound today of cases where high electricity prices observed during situation of tight generation capacity are immediately qualified as exercises of market power. This justifies testing two different caps like 250 €/Mwh or 1000 €/Mwh. These values are definitely too low if the idea is to charge anything like a Value of Lost Load during curtailment. But they may reflect what Regulator or Competition Authorities find reasonable when capacity is tight. Wether one relies on a capacity or energy only market, and in this latter case whether the cap is set at 250 €/Mwh or 1000 €/Mwh, one can expect that uncertainty will have an impact on investors. It is thus interesting to compare the result on investments of these different mix of assumption: market design (capacity or energy only), regulation(price cap level) and attitude towards risk (P or CVaR driven Q). These different effects are illustrated on Tables 9 and 10 that describe results obtained with price caps of 1000 and 250 €/Mwh respectively.

	Coal	CCGT	OCGT	Total	Capacity	Hours
CM/RN	15527	64472	6000	86000	0	0
CM/RA	15214	64472	6000	86000	0	0
EO/RN	15530	64470	3000	83000	3000	10
EO/RA	15217	64453	0	80000	6000	50

Table 9: Price cap: 1000 €/Mwh

	Coal	CCGT	OCGT	Total	Capacity	Hours
CM/RN	15527	64472	6000	86000	0	0
CM/RA	15377	55098	15525	86000	0	0
EO/RN	15546	64454	0	80000	6000	50
EO/RA	15232	44906	19862	80000	6000	50

Table 10: Price cap: 250 €/Mwh

Both tables exhibit the same structure: CM and EO respectively refer to models embedding Capacity Markets or based on Energy Only. RM and RA respectively designate simulations conducted with Risk Neutral agents (zero risk premium) or Risk Averse (CVaR driven Q). The three first columns indicate the invested capacity (in MW) of coal, CCGT and OCGT respectively. The fourth column "Total" gives the total invested capacity: anything less than 86000 signals curtailment. The two latter columns describe the curtailment in two different ways: "Capacity" indicates the shortage of capacity with respect to the peak; "Hours" is the number of hours of curtailment. The message conveyed by these two tables is rather straightforward. The capacity market applied in an environment where there is no demand uncertainty guarantees that demand is met at all times: the capacity value ν gives the adequate additional incentive to invest. One also finds identical generation systems when the risk premium is null, whether the price cap is 1000 or 250 €/Mwh. Things change however when agents become averse to risk (when the risk premium is different from zero). Decisions of risk averse agents no longer optimize the generation system in case of a low price cap: they shift capacity from CCGT to OCGT in response to the low energy price. This reminds of the period in the US where investors subject to an obligation to serve shifted investments towards gas turbine in response to the uncertainty created by prudence reviews. Not surprisingly the energy only market leaves some demand unsatisfied. Joskow (2007) describes that effect; an energy only market invests in so far as the price charged during curtailment induces it do so. A too low price compared to the VOLL implies a market failure. The new result here compared to Joskow (2007) is that uncertainty, not unexpectedly enhances the impact of the market failure. The shortage goes from 3000 to 6000 Mw and its duration from 10 to 50 hours when moving from a risk neutral to a risk averse world. The situation is worse when the cap is limited to

250 €/Mwh. The curtailment amounts to 6 GW during 50 hours whether in a risk neutral or risk averse world. It combines however with a desoptimization of the generation system in an energy only market where the price cap is limited to 250 €/Mwh.

7 Financial interpretation and extension

7.1 Risk premium

The above discussion commonly referred to zero and non zero risk premium without really elaborating or quantifying the non zero risk premium. The notion of a zero risk premium when all agents are risk neutral and base their decision on the sole expected value of the profit is readily understood. Investors are indifferent to the profile of the profit; they are even indifferent as to whether some of these scenarios would lead to bankruptcy: they only look at the average; their risk premium is null. We mentioned a non zero risk premium for risk averse investors behaving according to a risk neutral probability Q but did not give much precision. We shall now explicitly compute the risk premium embedded in the risk neutral probabilities Q found in the different simulation. We do this by first calling on standard notions of corporate finance.

The industry commonly relies on CAPM based methods for capacity expansion. This standard theory (see any corporate finance textbook) commonly refers to the "Sharpe ratio" computed on the basis of the "market line". The Sharpe ratio of the market, or of a sector is defined as follows: let a unit investment taking place in year 0 and giving a random payoff R in period 1. Let $E(R)$ and $\sigma(R)$ be respectively the expectation and standard deviation of this random payoff. Let also R_f be the payoff of a unit investment at the risk free rate. The Sharpe ratio, or the premium required by the sector to compensate the market for its risk is equal to

$$RP \equiv \frac{E(R) - R_f}{\sigma(R)}.$$

One can compute a similar risk premium for the capacity expansion obtained by assuming a CVaR behavior of the investors. Needless to say the computation only makes sense for the RA simulation cases, as all RN cases have zero risk premium. In order to illustrate the point we report some relevant results from the simulation such as the total investment cost, and the net margins accruing in each scenario. Using the same risk aversion ($\alpha(k) = 0.9$) for all technologies we compute Sharpe ratios for the total investment.

Table 11 reports these results for the different RA simulation runs as well as the resulting risk premium.

	Investment	Expected net margin	Standard deviation	Risk premium
1000/CM/RA	8002178	95214	4118822	2.31%
1000/EO/RA	7643695	321843	4492441	7.16%
250/CM/RA	7799639	93920	4004185	2.35%
250/EO/RA	7221351	382413	3055292	12.5%

Table 11: computation of risk premium

One sees that the capacity market drastically reduces the return demanded by investors to compensate for risk. This is the financial interpretation of the pattern described in Section 6. Capacity markets decrease the risk premium and hence entice investments. The table also suggests a way to calibrate the risk function (choosing the α) used in the model. Suppose that one observes a certain risk premium on the market or a sudden change of risk premium (as observed on the bond market in the course of the development of the subprime crisis), then one can select a value of α such that the CVAR_α behavior reproduces the desired risk premium.

7.2 Extension: different risk aversions

The above discussion immediately suggests an extension of the above model. We further reason that, as different technologies are subject to different fuel and CO_2 risks, they may also be subject to other risks not explicitly represented in the model. Stoft (2002) explains the particular demand risk that affects open cycle gas units. Risks of disruptions are also commonly mentioned for gas supply in Europe. For these different reasons, one may want to assume different risk attitudes of investors with respect to different technologies. An investor may be more risk averse when investing in a gas plant than in a combined cycle unit. The above formulation can be readily modified to accommodate this extension.

Suppose that investments into Coal, CCGT and OCGT are realized by different agents each behaving according to a different CVAR(k) behavior. In other words, suppose that the investors in plants k and k' refer to different α for these different plants. Consider an investor in technology k. As shown in Section 5.3, this investor receives a net margin

$$\text{margin}(k; f, n, b) \equiv \big[\sum_{\ell \in L} \tau(\ell)\mu(k; \ell, f, n, b) + \nu + a(k, b)\lambda(f, n, b) - I(k)\big]. \quad (38)$$

Conducting the same reasoning as before, one associates a risk neutral probability $\phi(k; f, n, b)$ to this investor by solving the following complementarity problem

$$0 \leq u(k; f, n, b) - \text{VAR}(k) + \text{margin}(k; f, n, b) \perp \phi(k; f, n, b) \geq 0 \quad (39)$$

$$0 = -\sum_{f \in F, b \in B, n \in N} \phi(k; f, n, b) + 1 \quad (40)$$

$$0 \leq -\phi(k; f, n, b) + \frac{1}{\alpha(k)} pf(f)pn(n)pb(b) \perp u(k; f, n, b). \quad (41)$$

The investment criterion of player k can thus again be written as

$$0 \leq \sum_{f \in F, n \in N, b \in B} \phi(k, f, n, b)\text{margin}(k, f, n, b) \perp x(k) \geq 0. \quad (42)$$

The only difference with the preceding model lies in $\alpha(k)$ in relation (41).

Table 12 illustrates an assumption where investors are progressively less risk averse as we go from open cycle gas turbine to combined cycle gas turbine and to coal. Tables 13 and 14 report the corresponding results for the two price caps.

α Coal	α CCGT	α GT
1	.7	0.5

Table 12: Risk aversion different technologies

	Coal	CCGT	OCGT	Total	Capacity	Hours
CM/RA	15527	64473	6000	86000	0	0
EO/RA	15546	64454	0	8000	6000	50

Table 13: Price cap: 1000 €/Mwh

	Coal	CCGT	0CGT	Total	Capacity	Hours
CM/RA	15214	44942	25844	86000	0	0
EO/RA	15989	44011	0	60000	26000	360

Table 14: Price cap: 250 €/Mwh

While the high price cap (1000 €/Mwh) maintains the same degree of reliability as in the case with technology independent α (see Table 9), one observes a dramatic degradation of this reliability when the price cap is lowered to 250 €/Mwh (Table 14) compared to the case with technology independent α (see Table 10).

7.3 Extension: a single corporate discount rate

The treatment of Section 5.4 and 7.2 uses technology specific discount rates. This corresponds to a relatively recent practice of investment banks where different plants are discounted at different rates. Having several different risk neutral probabilities for non traded risks is also perfectly compatible with finance theory. A probably more common practice is to discount all equipments at a single rate that corresponds to the one of the company. We now consider this alternative approach where we assume in the context of this example a single company of assets $x(k)$ behaving perfectly competitively. Its exposure to risk, and hence, the risk premium depends on $x(k)$. We now want to find both $x(k)$ and the risk premium. Let margin $(k; f, n, b)$ be defined as before, whether for an energy only or capacity market model (relations (36)and (37)). The risk profile associated to a unit capacity of the company is given by

$$\left(\sum_{k\in K} x(k)\right) \text{margin}(x; f, n, b) = \sum_{k\in K} \text{margin}(k; f, n, b)x(k). \tag{43}$$

Applying the CVaR complementarity condition to this margin, we can now readility convert the P version of the energy only and capacity market models to a corresponding Q version that represents risk averse, CVaR behaving agents operating with a single discount rate. Expressing that these agents maximize their profit by investing according to the x dependent risk neutral probability $\phi(x; f, n, b))$ derived from the duality contitions of the CVaR, we introduce in the model the following conditions :

$$0 \leq u(x; f, n, b) - \text{VaR} + \text{margin}(x; f, n, b) \perp \phi(x; f, n, b) \geq 0 \quad (44)$$

$$0 = - \sum_{f \in F, b \in B, n \in N} \phi(x; f, n, b) + 1 \quad (45)$$

$$0 \leq -\phi(x; f, n, b) + \frac{1}{\alpha} pf(f) pn(n) pb(b) \text{margin}(x; f, n, b) \quad (46)$$
$$\perp u(x; f, n, b) \geq 0$$

We replace the investment criteria (21) (energy only market) and (23) (capacity market) by the single investment criterion expressed as

$$0 \leq - \sum_{f \in F, b \in B, n \in N} \phi(x; f, n, b) \text{margin}(k; f, n, b) \perp x(k) \geq 0 \quad (47)$$

where $\text{margin}(k; f, n, b)$ is equal to

$$\begin{aligned} \text{margin}(k; f, n, b) \quad & \equiv \sum_{\ell \in L} \tau(\ell) \mu(k; \ell, f, n, b) + \nu \\ & + a(k, b) \lambda(f, n, b) - I(k) \end{aligned} \quad (48)$$

for the capacity market and

$$\begin{aligned} \text{margin}(k; f, n, b) \quad & \equiv \sum_{\ell \in L} \tau(\ell) \mu(k, \ell, f, n, b) \\ & + a(k, b) \lambda(f, n, b) x(k) - I(k) \end{aligned} \quad (49)$$

for the energy only market. The rest of the model (that is relations (16) to (20)) remains unchanged.

The numerical properties of this model are particularly bad. The high non-linearity in the calculation of the margin leads to multiple solutions which signal a highly non convex model. This suggests an underlying bad economics: imposing a single risk neutral measure to plants exposed to different risk profiles is unlikely to be correct if these risks are not traded. As is often the case "simplifying" the market may make it incomplete and create other problems. We illustrate this phenomenon for the case of a price cap of 1000 €/Mwh in

Table 15. The two calculations for the capacity markets exhibit the same investment pattern but a very different return due to different hourly prices. For the energy only market we obtained different investment mixes.

	Coal	CCGT	OCGT	Total	Capacity	Hours	Expected return
CM/RA	15524	44942	25844	86000	0	0	292054
CM/RA	15524	44942	25844	86000	0	0	72489
EO/RA	15216	44936	22847	83000	3000	10	69765
EO/RA	15232	44906	19862	80000	6000	50	322338

Table 15: Price cap: 1000 €/Mwh

8 Conclusions

Capacity expansion in the traditional regulated power industry was a technically easy problem. The model could be cast in an optimization form and hence benefit from all the advances of the area. Uncertainties were reasonably mild and could be passed in average form to consumers. Households that only allocate a small fraction of their income to electricity did not suffer much. Large industrial consumers were not facing yet the fierce competition brought by globalization: long term demand was thus relatively inelastic. Because the monopoly regime protected utilities, their business was essentially risk free which allowed for an easy financing of the huge capital that some technologies required. Restructuring and environmental concerns have change all of this. The optimization model no longer fits the description of a market with a set of companies in competition; household are encouraged to switch suppliers and large industrial consumer threaten to relocate part of their activities to more industry friendly but also environment hostile regions. Risk is now piling up from everywhere, whether as a result of world movement or from what Hogan called "reforms of reforms". And last but not least, we are not too sure that the market sends the right signals to invest.

In order to look at this novel situation, we try to move the old technique of capacity expansion models into this new environment. Equilibrium substitutes optimization to account for competition; because of missing information we stick to the former fixed demand model. We also explicitly introduce an uncertain environment. Questions of cost of capital should then come back to the forefront as these new risks must have had an impact on CAPM's β. Unfortunately, the short time elapsed since the beginning of restructuring and its evolving nature make it difficult to calibrate the standard β coefficient of the traditional CAPM with a reasonable degree of certainty. We therefore bypass the standard approach and model the impact of risk through risk functions that we take here as CVaR. In order not to wander in unknown territories, we illustrate however how this approach allows one to fall back on CAPM language. Whatever information the market can tell us about notions such as the Sharpe ratio can then be used to calibrate our CVaR function. It is also interesting that all this allows one to accommodate different assumptions of incentives to invest.

Models such as these can be used to assess policies that are still in discussion today. Among them the incentive to investment in new capacities are crucial as a lack of incentive could damage security of supply. Not surprisingly we find that a capacity market is better suited than an energy only market, at least if regulatory authorities are induced to set low cap on the prices charged when capacity is tight. We also find that uncertainty plays an important role. It enhances the shortcomings of energy only system. It also adds an additional drawback by desoptimizing the generation system.

The paper overlooks many issues that we briefly mention here. The first one is that the discussion is conducted on a very small example. This may be justified for pedagogical reasons but it also raises questions. We only briefly elaborated on possible numerical difficulties but we can put them in context here. All equilibrium problems discussed here are non convex. This begins with the introduction of free allowances; it continues with the move to risk averse investors and the introduction of a risk neutral probability. It extends to the case of a simple risk neutral probability that applies to all technologies. This non convexity should be studied. A second point is that the model is static or, in the parlance of stochastic programming, two stage. Extending a two-stage stochastic program to a multistage one is not conceptually difficult with risk neutral investors. But the matter is different with risk averse agents. Artzner et al. (2002) and (2004) discuss the problem of coherent multiperiod risk functions. Last but not least, the EU-ETS evolves. The small example that serves a a basis of discussion corresponds to the 2007 context. Discussions on an update of the law are in progress, and what we know from it foresees a new rich area of uncertainties.

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